OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-5 : MAXIMA AND MINIMA

UNIT TEST-1

1. If a_α is the greatest term in the sequence $a_n = \dfrac{n^3}{n^4+147}$, $n = 1, 2, 3, \ldots$, then α is equal to ___________.

2. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi– vertical angle is tan−1 $\dfrac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is.

3. The sum of the maximum and minimum values of the function $f(x) = [5x - 7] + [x^2 + 2x]$ in the interval $\left[\dfrac{5}{4}, 2\right]$, where [t] is the greatest integer $\le t$, is ___________.

Hints and Solutions

1. (5)

Let $$y = \frac{x^3}{x^4+147} = f(x)$$

For increasing function

$$\frac{dy}{dx} > 0$$

$$\frac{-x^2(x^4-441)}{(x^4+147)^2} > 0 \Rightarrow x^4 < 441$$

$\because$ $x^4 < 441$

For maximum/minima $\dfrac{dy}{dx} = 0$

$\Rightarrow$ $x^4 = 441,$

$\Rightarrow$ $x = \alpha,\ 4 < \alpha < 5$

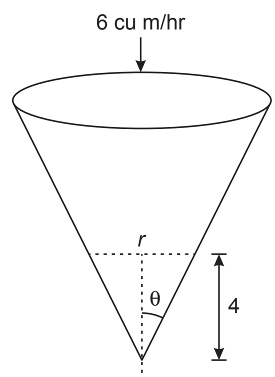
$\Rightarrow$ Maximum value of $f(x)$ is at $x = 4$ or $x = 5$

$$f(4) = \frac{64}{403},\ f(5) = \frac{125}{772}$$

$\because$ $f(5) > f(4)$

$\Rightarrow$ $\alpha = 5$

2. (5)



$\tan\theta = \dfrac{3}{4}$

$$v = \frac{1}{3}\pi r^2 h \qquad \ldots\text{(i)}$$

And $$\tan\theta = \frac{3}{4} = \frac{r}{h} \qquad \ldots\text{(ii)}$$

i.e. if $h = 4$, $r = 3$

$$v = \frac{1}{3}\pi r^2\left(\frac{4r}{3}\right)$$

$$\frac{dv}{dt} = \frac{4\pi}{9}3r^2\frac{dr}{dt} \Rightarrow 6 = \frac{4\pi}{3}(9)\frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{2\pi}$$

Curved area $= \pi r\sqrt{r^2 + h^2}$

$$= \pi r\sqrt{r^2 + \frac{16r^2}{9}} = \frac{5}{3}\pi r^2$$

$$\frac{dA}{dt} = \frac{10}{3}\pi r\frac{dr}{dt}$$

$$= \frac{10}{3}\pi\cdot 3\cdot\frac{1}{2\pi}$$

$$= 5$$

3. (15)

$f(x) = |5x - 7| + [x^2 + 2x]$

$= |5x - 7| + [(x + 1)^2] - 1$

Critical points of

$f(x) = \frac{7}{5}, \sqrt{5}-1, \sqrt{6}-1, \sqrt{7}-1, \sqrt{8}-1, 2$

∴ Maximum or minimum value of $f(x)$ occur at critical points or boundary points

∴ $$f\left(\frac{5}{4}\right) = \frac{3}{4} + 4 = \frac{19}{4}$$

$$f\left(\frac{7}{5}\right) = 0 + 4 = 4$$

as both $|5x - 7|$ and $x^2 + 2x$ are increasing in nature after $x = \frac{7}{5}$

∴ $f(2) = 3 + 8 = 11$

∴ $f\left(\frac{7}{5}\right)_{\min} = 4$

and $f(2)_{\max} = 11$

Sum is $4 + 11 = 15$